# SPH4U: Energy and Momentum Review Solutions

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|---|---|---|---|
| $\vec{p} = m\vec{v}$ | $\Delta\vec{p} = \vec{F}\Delta t = m\Delta\vec{v}$ | $W = F\Delta d\cos\theta$ | $F = \lvert kx \rvert$ |
| $E_k = \frac{1}{2}mv^2$ | $E_e = \frac{1}{2}kx^2$ | $E_g = mgh$ | $W = \Delta E_k$ |
| $R_{earth} = 6.38 \times 10^6$ m | $M_{earth} = 5.98 \times 10^{24}$ kg | $G = 6.67 \times 10^{-11}$ $Nm^2/kg^2$ | $E_g = -\dfrac{GMm}{r}$ |

1. How much work needs to be done to lift a 5.0 x $10^2$ kg satellite from the surface of the Earth to a height of 200 km above the surface? How fast does the satellite need to be launched from the earth so that it reaches this height before falling back to earth?

Work to be done = change in gravitational potential energy = $E_g(r_{earth} + 200km) - E_g(r_{earth})$

$= -\dfrac{GMm}{r_{earth} + 200\times10^3} - \left[-\dfrac{GMm}{r_{earth}}\right]$ = $9.5 \times 10^8$ J

Satellite must have no kinetic energy when it reaches a height of 200 km above the surface, so by conservation of energy:
$E_g$(surface of earth) + $E_k$(surface of earth) = $E_g$(200 km above surface), so
$E_k$(surface of earth) = $E_g$(200 km above surface) - $E_g$(surface of earth) = $9.5 \times 10^8$ J
$\dfrac{1}{2}mv^2 = 9.5 \times 10^8$ J, re-arrange and solve for speed, $v = 1.9 \times 10^3$ m/s

2. A 19 kg curling stone is sliding along the ice at 4.6 m/s [E 10° S] when an errant hockey puck (170 g) traveling at 20 m/s [N 55° E] strikes it. After the collision, the curling stone continues on at 4.61 m/s [E 6.56° S]. Determine the velocity of the hockey puck immediately after the collision.

Indicate a frame of reference (e.g. north = positive y, east = positive x)
Use conservation of momentum in both x and y directions:

x direction: $p_{1x} + p_{2x} = p'_{1x} + p'_{2x}$ so $m_1v_{1x} + m_2v_{2x} = m_1v'_{1x} + m_2v'_{2x}$
y direction: $p_{1y} + p_{2y} = p'_{1y} + p'_{2y}$ so $m_1v_{1y} + m_2v_{2y} = m_1v'_{1y} + m_2v'_{2y}$

m1 = 19 kg, m2 = 0.170 kg
$v_{1x} = 4.6\cos 10°$, $v_{1y} = -4.6\sin 10°$ and $v'_{1x} = 4.61\cos 6.56°$, $v'_{1y} = -4.61\sin 6.56°$
$v_{2x} = 20\sin 55°$, $v_{2y} = 20\cos 55°$

Solve for $v'_{2x}$ and $v'_{2y}$ separately and then find the magnitude of the resulting vector using Pythagoras
($\sqrt{v_{2x}^2 + v_{2y}^2}$, and the angle by $\tan^{-1}\left(\dfrac{v_{2y}}{v_{2x}}\right)$

3. You apply a horizontal force of 500 N to push around a shopping cart at a constant speed of 1.5 m/s for an hour. (a) How much work have you done? (b) How much work was done on the shopping cart?

$W = F\Delta d\cos\theta$ = (500 N)(1.5 m/s × 3600 s)(cos 0°) = $2.7 \times 10^6$ J is the work done by you

Since the cart goes at constant speed, there must be work done by friction in the opposite direction, and because the work done on an object is defined as $W = \Delta E_k$, there is no change in kinetic energy and therefore no work don on the cart

4. In yet another mishap in his misguided mission to make a meal of the Roadrunner, Wile E. Coyote (50 kg) flies horizontally through the air towards a brick wall. A conveniently placed spring (k = 5000 N/m) compresses 0.75 m and brings him to a complete stop in 0.45 s. (a) What is the maximum force exerted by the spring on the coyote? (b) Assuming that the force exerted by the spring increases linearly with time, sketch a graph of the spring force as a function of time from first contact until the coyote comes to a stop. (c) Using your graph, find the coyote's change in momentum. (d) What was the coyote's speed when he first came in contact with the spring?

Maximum force exerted by the spring: $F = |kx|$ = (5000 N/m)(0.75 m) = 3750 N

The graph is linear, starting with F = 0 N at 0 s, and increases to F = 3750 N at 0.45 s

The area under the graph is the impulse $F\Delta t$, which is the change in momentum. The area is the area of a triangle $\frac{1}{2}bh$ with a base of 0.45 s and a height of 3750 N. The coyote is brought to a stop, so the change in momentum is negative ($-8.4 \times 10^2$ kg m/s)

To find the coyote's speed, use $\Delta\vec{p} = m\Delta\vec{v}$, so $-8.4 \times 10^2$ kg m/s = (50 kg)(0 – $v$) since the final velocity is 0, and $v = 1.7 \times 10^1$ m/s.

5. A certain amphibian (25 kg) jumps vertically up off a platform 3.5 m high with an initial speed of 2.0 m/s, but fortunately lands on (sticks to) a 50 kg giant marshmallow just above the ground. The marshmallow is attached to a spring, which compresses 20 cm before the amphibian is brought to a stop.

**Using energy and momentum techniques only,** determine (a) the speed of the amphibian just before contact with the marshmallow, (b) the speed of the amphibian and marshmallow immediately after impact, and (c) the spring constant (in N/m). Don't forget that there is also a change in gravitational potential energy when the spring compresses!

Conservation of energy:

$E_k + E_g$ (at the top) = $E_k$ (at the bottom), assuming Eg = 0 at the bottom

$\frac{1}{2}mv_1^2 + mgh = \frac{1}{2}mv_2^2$, and solve for $v_2$ = 8.5 m/s

Conservation of momentum for speed of amphibian and marshmallow after impact:

$m_1v_1 + m_2v_2 = (m_1 + m_2)v_3$

$(25)(8.5) + (50)(0) = (25 + 50)v_3$, speed = 2.8 m/s

Conservation of energy to find the spring constant:

$E_k + E_g$ before spring compression = $E_e$

$\frac{1}{2}mv^2 + mgh = \frac{1}{2}kx^2$

$(0.5)(75)(2.8)^2 + (75)(9.8)(0.2) = (0.5)k(0.20)^2$

Solve for k, $k = 2.2 \times 10^4$ N/m